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A NURBS APPROXIMATION OF EXPERIMENTAL STRESS-STRAIN CURVES

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ABSTRACT

A compact universal representation of monotonic experimental stress-strain curves of metals and alloys is proposed. It is based on the nonuniform rational Bezier splines (NURBS) of second order and may be used in a computer library of materials. Only six parameters per curve are needed; this is equivalent to a specification of only three points in a stress-strain plane. NURBS-functions of higher order prove to be surplus. Explicit expressions for both yield stress and hardening modulus are given. Two types of curves are considered: at a finite interval of strain and at infinite one. A broad class of metals and alloys of various chemical compositions subjected to various types of preliminary thermo-mechanical working is selected from a comprehensive data base in order to test the methodology proposed. The results demonstrate excellent correspondence to the experimental data.

Keywords: work hardening, stress-strain curve, spline approximation, nonuniform rational B-spline, NURBS.

INTRODUCTION

Metal forming is a basic operation in many technological chains but it is restricted by two factors: maximum load on a tool of machine and generation of defects (micro-, meso-, and macro-cracks) inside a work piece. In order to overcome the restrictions, a roll stamping technique (RS) is developed in Oryol State University – Education-Science-Production Complex (Russia) [1, 2]. The RS is based on the effects of severe plastic deformation and compressive stresses created in the course of the process and can be used not only in the metal forming field but also in surface hardening technology [3-10] and for creation of micro- and nano-structures [11-14]. The value of plastic deformation (i.e. the Odkvist's parameter) can reach ≈10 or higher under RS working. Therefore the design of such a process is an extremely complicated task that cannot find a solution without a computer. This paper is dedicated to the particular aspect of numerical modeling – a universal compact representation of experimental stress-strain curves of metals and alloys at great and super great strain.

At the current state of physics, it is impossible to calculate analytically the stress-strain dependence of a uniformly deformed material (i.e. on the ground of the fundamental principles only). Hence, it is necessary to exploit any phenomenological approximation of the experimental data. Many hardening laws were proposed [15]. They may be successful in particular cases but none of them proved to be universal. For example, the well known power law

$$\sigma = A\varepsilon^n + B$$

is not consistent with the other well known exponential one

$\sigma = A\varepsilon + B - Ce^{-k\varepsilon}$

Therefore, experimental data should be represented in a primary graphic form and then it has to be approximated by an appropriate hardening law. But, each comprehensive material library (for example www.matdat.com system – an online data base) comprises more than a thousand of stress-strain curves and there is no guarantee of finding a sufficiently exact approximation of a specific experimental curve. In such a case, the only valid representation is the "point by point" one. But, the plastic deformation real value may exceed at super great strain the experimental range when measuring the stress-strain curve and the information about the asymptotic behavior at infinity proves to be lost (note that some modern procedures permit to measure the deformation behavior at super great strain [16] but they are not widespread yet).

On the other hand, taking into account the need of computer calculations, an electronic data base should be connected to a program code. To make this task easy, a universal compact representation is extremely desirable. It should be pointed out that such a program code may be intended not only for metal forming problems; the stress-strain dependence may be also a source of information on the material constants [17 - 19].

To resolve the problems above, the nonuniform rational B-splines of second order were proposed [20, 21]. They need only 6 parameters per a curve, which is an equivalent to a specification of 3 points in the stress-strain plane. In this work, the main results of those papers are reproduced and the proposed approximation is tested on a broad class of materials.

EXPERIMENTAL

There are some reasons for the widespread acceptance and popularity of NURBS in the CAD/CAM and graphics community [22]. Despite some drawbacks, NURBS are considered an optimal choice in many tasks. Other free-form schemes such as Bezier, Coons, and Gordon exhibit the same problems [22].

The curve reconstruction problem is studied in [23] where space curves of a free form are approximated using NURBS-snakes and a quadratic programming approach. As for the particular case of experimental stress-strain curves, this methodology may be considerably simplified. In this work, the NURBS-functions of a second order are proposed, which need only 6 parameters per a curve. Turn to the technical details. Let $\mathbf{p} = \mathbf{p}(u)$, $0 \le u \le 1$ be a vector-function in $\varepsilon\sigma$ plane of the form

$$\mathbf{p}(u) = \frac{h_0\mathbf{p}_0N_{0,3}(u) + h_1\mathbf{p}_1N_{1,3}(u) + h_2\mathbf{p}_2N_{2,3}(u)}{h_0N_{0,3}(u) + h_1N_{1,3}(u) + h_2N_{2,3}(u)} \tag{1}$$

where

$N_{0,3}(u) = (1-u)^2$, $N_{1,3}(u) = 2u(1-u)$, $N_{2,3}(u) = u^2$

are the Bernstein polynomials; $\mathbf{p}_0(0,\sigma_0)$, $\mathbf{p}_1(\varepsilon_1,\sigma_1)$, $\mathbf{p}_2(\varepsilon_2,\sigma_2)$ are the NURBS-function poles; σ_0 is the initial yield stress; (ε_2,σ_2) is the curve right endpoint; (ε_1,σ_1) is the point of intersection of two tangents to both curve endpoints. Weights h_0, h_2 are believed to be equal to 1 in order to guarantee that the NURBS curve passes via points $\mathbf{p}_0(0,\sigma_0)$ and $\mathbf{p}_2(\varepsilon_2,\sigma_2)$. Weight h_1 may be tailored visually or according to any criterion. So, only 6 parameters σ_0, ε_1, σ_1, ε_2, σ_2, h_1 are required to store the curve.

The hardening modulus formula (this modulus enters the Prandtl-Reuss flowing low) follows from eq. 1 on the ground of differentiation:

$$H' = \frac{d\sigma}{d\varepsilon} = \frac{d\sigma / du}{d\varepsilon / du} = \frac{P(u)}{Q(u)} \tag{2}$$

where $0 \le u \le 1$,

$$\begin{aligned}P(u) = {} & (\sigma_0N'_{0,3} + h_1\sigma_1N'_{1,3} + \sigma_2N'_{2,3}) \\ & (N_{0,3} + h_1N_{1,3} + N_{2,3}) - \\ & -(\sigma_0N_{0,3} + h_1\sigma_1N_{1,3} + \sigma_2N_{2,3}) \\ & (N'_{0,3} + h_1N'_{1,3} + N'_{2,3})\end{aligned}$$

$$\begin{aligned}Q(u) = {} & (h_1\varepsilon_1N'_{1,3} + \varepsilon_2N'_{2,3})(N_{0,3} + h_1N_{1,3} + N_{2,3}) \\ & -(h_1\varepsilon_1N_{1,3} + \varepsilon_2N_{2,3})(N'_{0,3} + h_1N'_{1,3} + N'_{2,3})\end{aligned}$$

and

$N'_{0,3}(u) = 2(u-1)$, $N'_{1,3}(u) = 2(1-2u)$, $N'_{2,3}(u) = 2u$

are the derivatives of the Bernstein polynomials. Both the yield law (eq.1) and the hardening one (eq.2) are given parametrically:

$\varepsilon = \varepsilon(u)$, $\sigma = \sigma(u)$, $H' = H'(u)$, where $0 \le u \le 1$.

But one can rewrite them in an explicit manner $\sigma = \sigma(\varepsilon)$ and $H' = H'(\varepsilon)$ aiming to obtain the reverse function $u = u(\varepsilon)$

from the dependence $\varepsilon = \varepsilon(u)$ in (1). This requires the solution of eq. (3):

$$Au^2 + Bu + C = 0 \tag{3}$$

where

$A = 2(\varepsilon - h_1(\varepsilon - \varepsilon_1) - \varepsilon_2$, $B = -2(\varepsilon - h_1(\varepsilon - \varepsilon_1)$, $C = \varepsilon$.

One can show [20] that only the left root of (3) is valid. Therefore,

$$u = \frac{\varepsilon - h_1(\varepsilon - \varepsilon_1) - \sqrt{h_1^2(\varepsilon - \varepsilon_1)^2 - \varepsilon(\varepsilon - \varepsilon_2)}}{2(\varepsilon - h_1(\varepsilon - \varepsilon_1) - \varepsilon_2} \tag{4}$$

The yield and hardening laws take the explicit form $\sigma = \sigma(u(\varepsilon)), H' = H'(u(\varepsilon))$ with the application of eq. (4).

Now, one can turn to the experimental curves, which exhibit a distinct linear asymptotics. The main task in such a case is to detect a critical point $\varepsilon = \varepsilon_a$ of the asymptotic region beginning (the inequality $\varepsilon_a < \varepsilon_2$ is required). Aiming this "reduced" strain and stress are introduced in correspondence with:

$$\tilde{\varepsilon} = \frac{\varepsilon}{\varepsilon + \varepsilon_1}, \ \tilde{\sigma} = \frac{\sigma - \sigma_0}{a\varepsilon + b - \sigma_0}$$

where $\sigma = a\varepsilon + b$ is the linear asymptotics of a curve under consideration. It turns out that if plane $\varepsilon\sigma$ has to be mapped onto the plane (rather onto the unit square [0;1]×[0;1]) $\tilde{\varepsilon}\tilde{\sigma}$ then the critical point $\tilde{\varepsilon} = \tilde{\varepsilon}_a$ reveals itself apparently. Then, it can be proceeded following the previous methodology eq.(1) - eq. (4) redefining ε_2 to ε_a and switching over the asymptotic regime $\sigma = a\varepsilon + b$ at $\varepsilon > \varepsilon_a$. Notice that the linear asymptotics is the simplest one and more complicated cases are out of the paper goals. Nevertheless, if the critical point $\varepsilon = \varepsilon_a$ of power like (or any other) asymptotics is detected then there is no problem to use the approach described above.

All the materials under investigation are selected from the comprehensive data base [24] according to the following criteria: 1) the stress-strain curve should be monotonic; 2) the initial yield stress should be given at zero plastic deformation; 3) the stress-strain curve should be "complicated" for approximation (it means that the hardening modulus alters considerably); 4) three types curves should be presented: without any distinct asymptotics, with a declined linear one, and with a horizontal one (more complicated power-like one is not considered); 5) if an approximation at infinite interval is needed, the experimental curve must contain a piece of the asymptotic region.

So, 20 metals and alloys of various chemical compositions subjected to various types of preliminary thermo-mechanical working are selected from the entire data base in order to test the approximation proposed.

RESULTS AND DISCUSSION

Logarithmic deformation ε or relative elongation (compression) q or shear deformation γ is used as a deformation measure depending on the situation. Yield stress is measured in [kg mm^{-2}] in all the figures excluding the latest three where the dimensionless reduced coordinates are exploited. The approximation results are presented directly in figures in order to minimize the paper size. The experimental data is shown by the points in Figs. 1 - 20 where the NURBS pole $\mathbf{p}_1$ and the tangents to $\mathbf{p}_0$ and $\mathbf{p}_2$ are also drawn. Figs. 1-6 do not obey any asymptotics. Figs. 7 - 20 possess an inclined asymptotics (7 - 14) or a horizontal one (15 - 20); they are obtained by the mapping of the initial experimental curves into the unit square $\tilde{\varepsilon}\tilde{\sigma}$ in order to detect the critical points (see above) but the proper curves are shown at the end of the paper where they are classified into 3 groups. The asymptotic regions in Figs. 7 - 20 are omitted. The type of the preliminary thermo-mechanical working is pointed out in the legend (in brackets). The material designation system is given together with the material name. The type of loading is absent in some figures due to its absence in the original data base [24].

The results presented above demonstrate an excellent accuracy of NURBS representation. Only 20 curves from the entire data base [24] are studied but their shapes vary in a wide range and the approximation results are trustworthy. Analyzing Figs. 1 - 20 one can see that second order NURBS-functions can guarantee an approximation of high quality and functions of higher order appear to be a surplus.

Figs. 21-23 show that curves 7-20 being transformed to reduced coordinates (eq. 5) are classified into 3 groups according to their shape (the curve number corresponds to that of the figure; curve No 9 is omitted): close to linear - Fig. 21, bell-shaped - Fig. 22, and a complex shape - Fig. 23. The classification is traceable in reduced coordinates only, whereas it is hided in the initial one. It is worth noting that the curves of different asymptotic

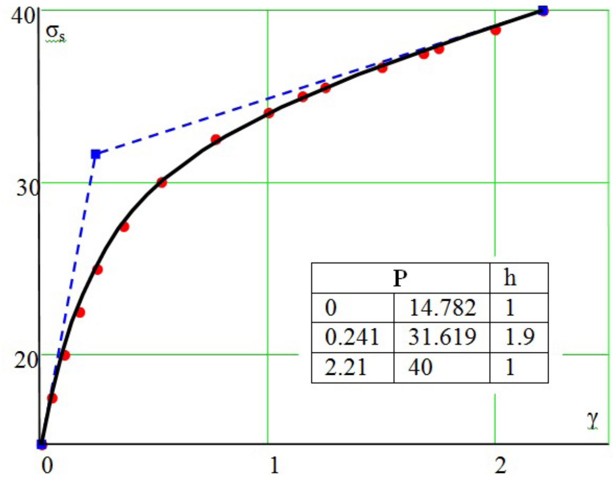


Fig. 1. Low carbon (C = 0,06 %) steel under torsion at a rate of 0,12 s^{-1} (t = 78˚C).

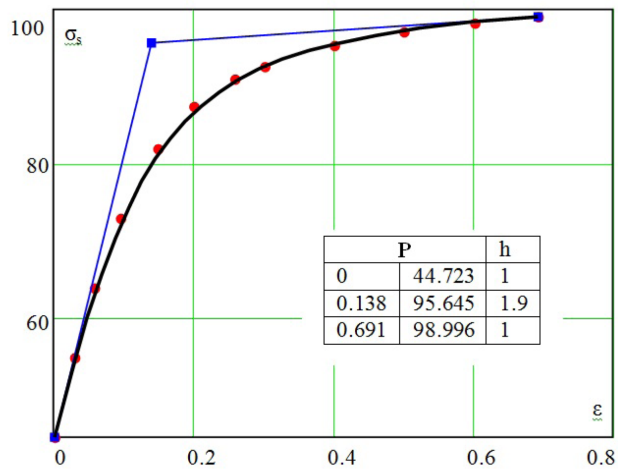


Fig. 4. Steel C45-DIN (annealing) under compression at a rate of 90 c^{-1} (t = 20˚C).

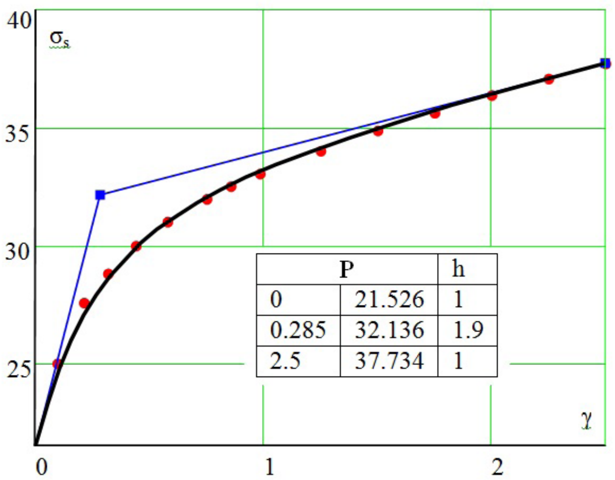


Fig. 2. The same at a rate of $2,4\times10^{-3}s^{-1}$.

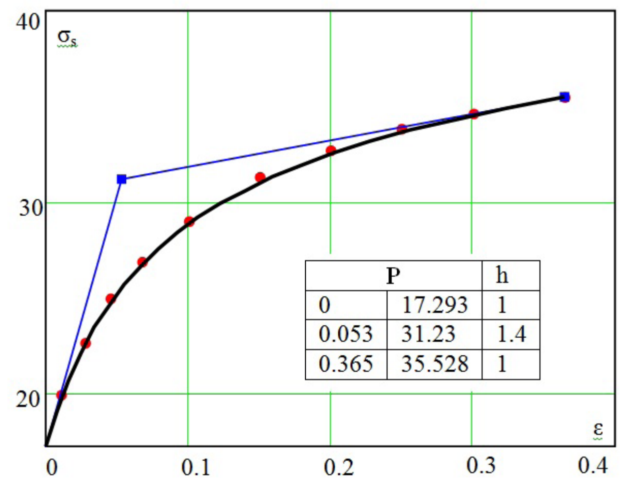


Fig. 5. Cu (cold upsetting up to ε = 10 %) under elongation (t = 20˚C).

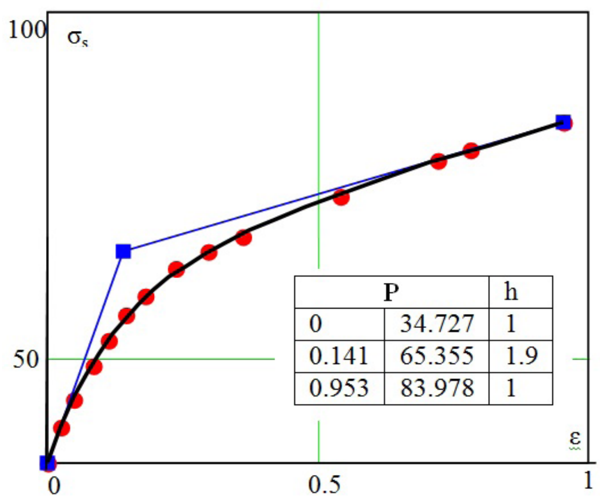


Fig. 3. Steel X10Cr13-DIN (annealing) under compression (t = 20˚C).

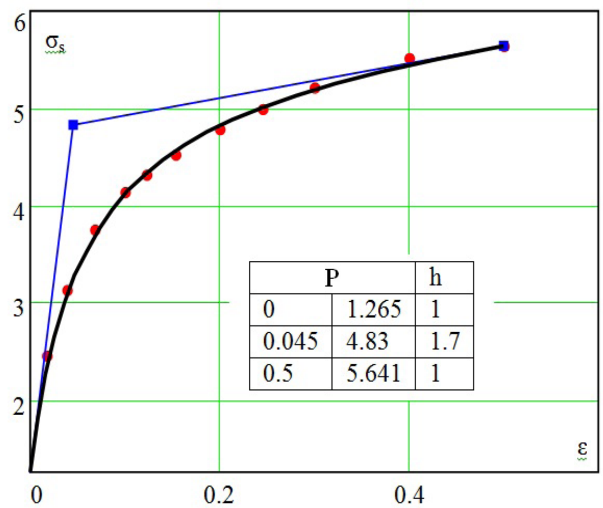


Fig. 6. Al (annealing, grain size 150 μm) under elongation (t=80˚C).

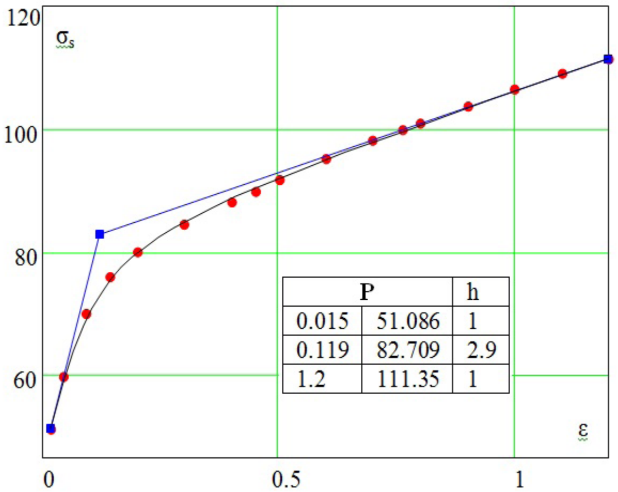


Fig. 7. Steel 45-GOST (hot rolling).

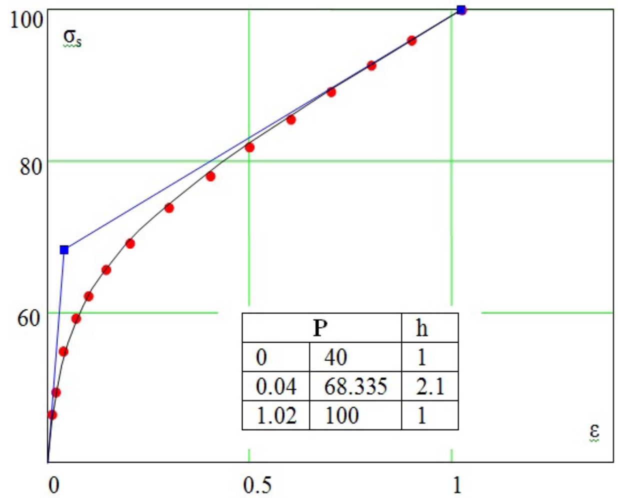


Fig. 10. Steel 30-GOST under elongaton (t = 20˚C).

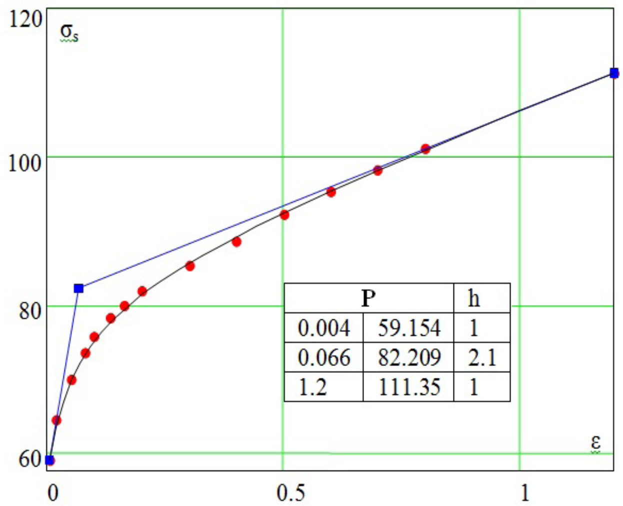


Fig. 8. Steel 45-GOST (drawing with cobbing up to ε = 0,13).

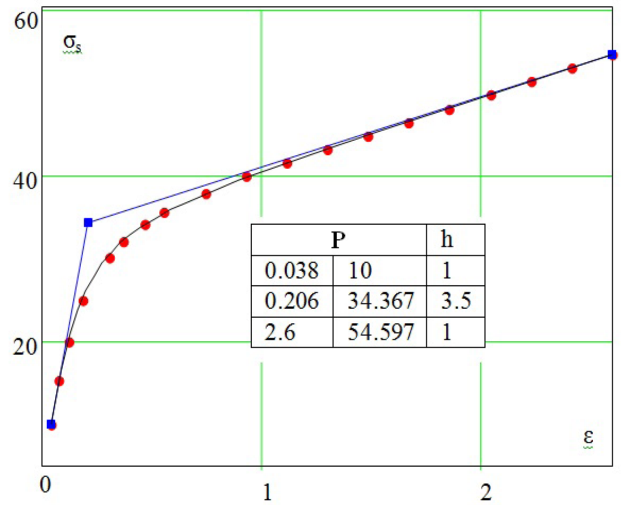


Fig. 11. Cu (annealing) under compression (t = 20˚C).

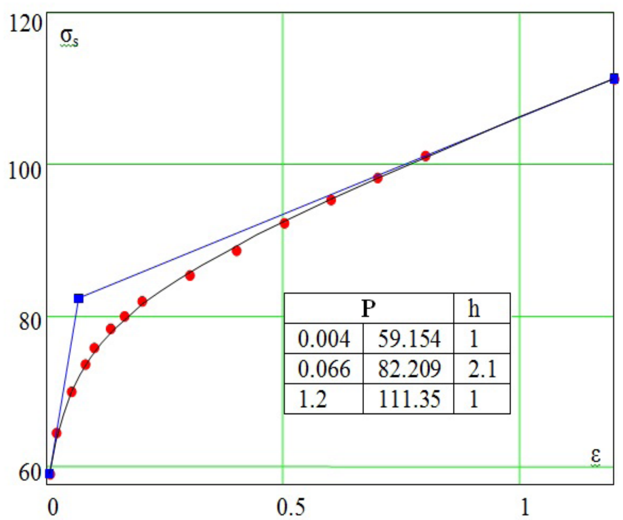


Fig. 9. Brass Л90-GOST under elongation (t = 20˚C).

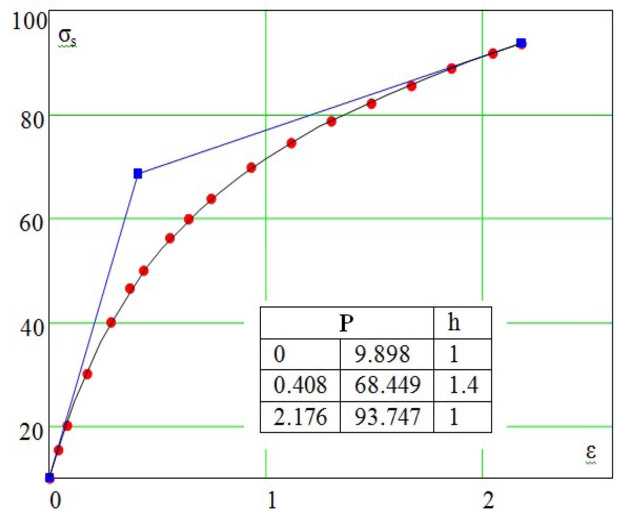
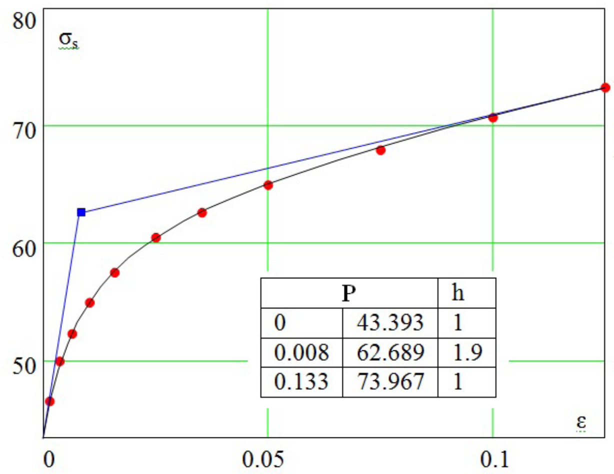
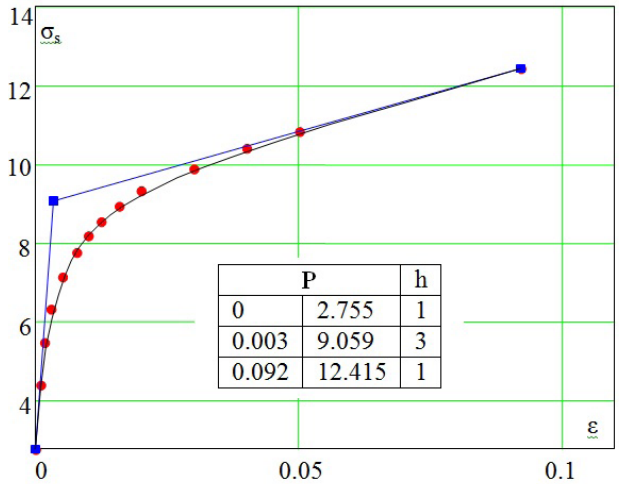


Fig. 12. Brass Л63-GOST (annealing) under compression (t = 20˚C).



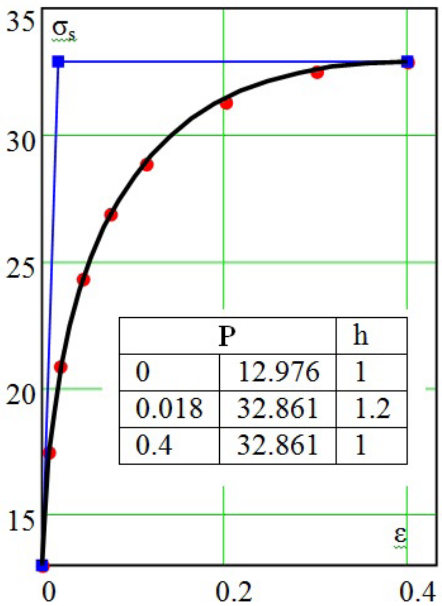
P		h
0	43.393	1
0.008	62.689	1.9
0.133	73.967	1

Fig. 13. Zr 99,7 % under elongation at a rate of $6,67\times10^{-5}s^{-1}$ (t = -196°C).



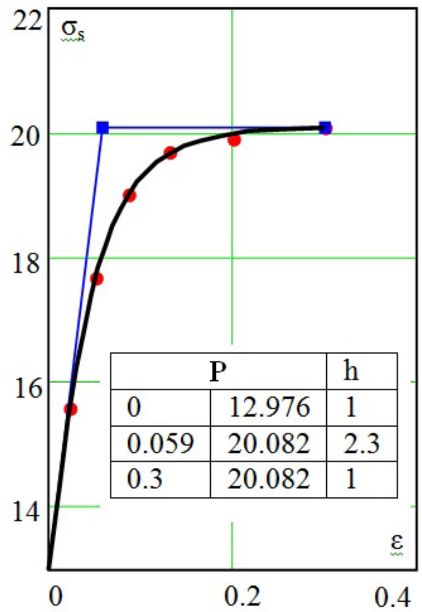
P		h
0	2.755	1
0.003	9.059	3
0.092	12.415	1

Fig. 14. Zn 99,995 % (pressing) under elongation (t = 20°C).



P		h
0	12.976	1
0.018	32.861	1.2
0.4	32.861	1

Fig. 15. Steel C45-DIN (annealing) under compression at a rate of 0,1 c-1 (t = 600°C).



P		h
0	12.976	1
0.059	20.082	2.3
0.3	20.082	1

Fig. 16. The same at t = 700°C.

behavior (inclined and horizontal) may appear in the same group; also, curves of similar asymptotic behavior may appear in different groups. The classification is grounded on the similar mechanism of plastic deformation of the different materials [25-29], but this issue demands a special study.

CONCLUSIONS

● NURBS-functions of second order can be used for representation of monotonic experimental stress-strain curves of metals and alloys.

● Only six parameters per curve are needed.

● The representation is found to be universal owing to its standard form.

● Both yield stress and hardening modulus are expressed from NURBS-representation in an explicit and easy manner.

● “Reduced” strain and stress were introduced in order to detect the beginning of linear asymptotic region for some curves.

● The 16 curves with the linear asymptotic behavior being transformed to the reduced coordinates are found

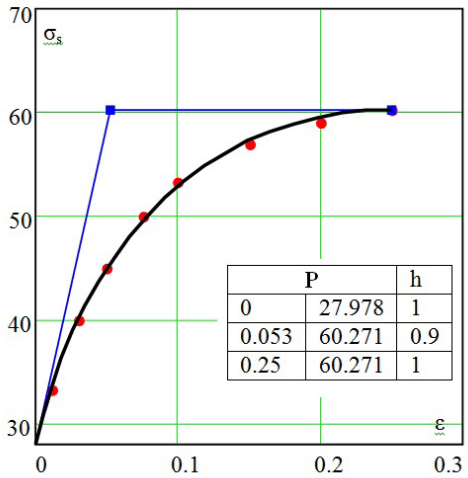


Fig. 17. Fe (upsetting up to ε = 5 %) under compression (t = 20˚C).

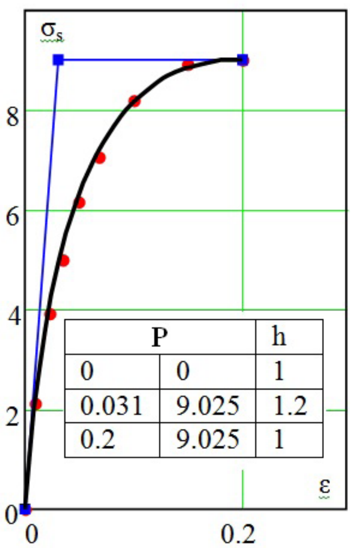


Fig. 18. Al alloy АБМц-GOST under compression at a rate of 5 m s-1 (t = 300˚C).

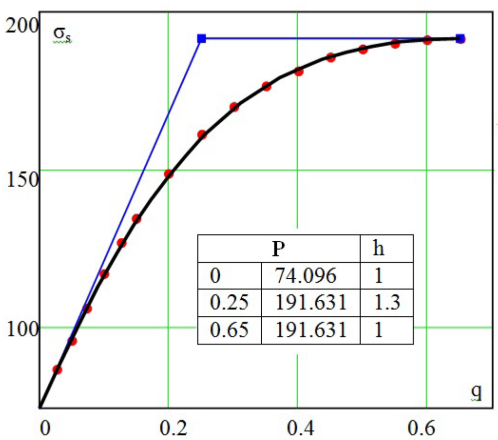


Fig. 19. Ni alloy ХН77ТЮ-GOST (hot rolling) under compression at a rate of 10 mm min-1 (t = 20˚C).

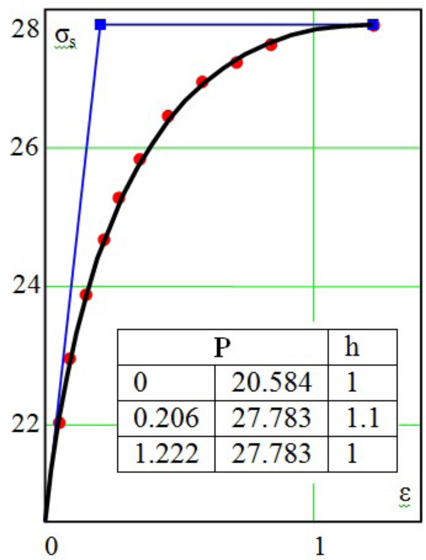


Fig. 20. Zn 99,9 % (annealing) under compression (t = 20˚C).

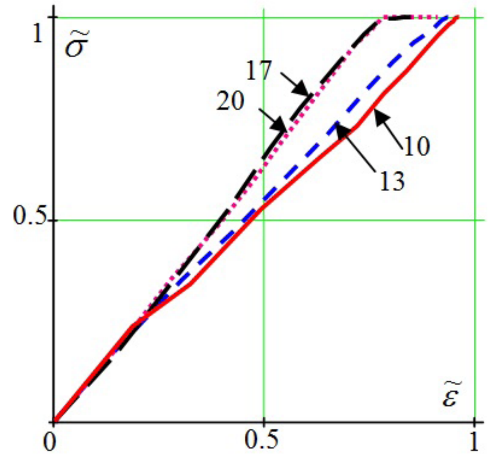


Fig. 21. Stress-strain curves 10, 13, 17, 20 in reduced coordinates.

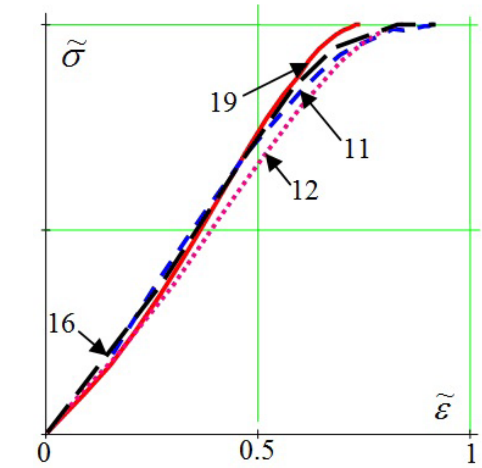


Fig. 22. Stress-strain curves 11, 12, 16, 19 in reduced coordinates.

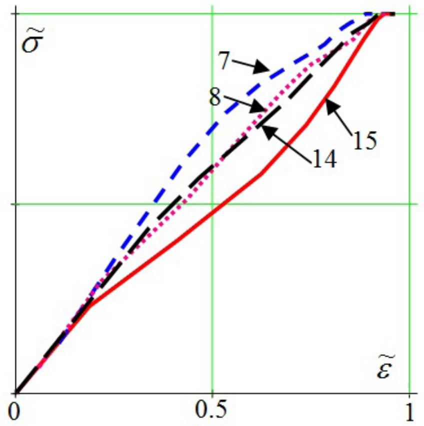


Fig. 23. Stress-strain curves 7, 8, 14, 15 in reduced coordinates.

to be classified by the three groups: close to linear shape, bell-shaped, and complex shape; this is conditioned by different mechanisms of plastic deformation in different groups.

● NURBS-functions of order three and more appear to be surplus.

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